TOLERANCE LIMIT ANALYSIS FOR ENGINEERING EXPRESSIONS

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TOLERANCE LIMIT ANALYSIS FOR ENGINEERING

EXPRESSIONS

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PREFACE

This study is concerned with the analysis of engineering type expressions containing variables with associated random errors. The primary objectives were to present methodology for approximating the mean and variance of an expression and through simulation gain some insight as to the form of the overall distribution of the expression. A computer program is used in the study to analyze several different expressions. Augmentation of the program is easily accomplished to facilitate the study of any desired expression.

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THE RESEARCH PROBLEM

Purpose

One aspect of statistical quality control deals with the statistics of combinations and tolerances. The problem associated with this study is the determination of the variance relationship for a function of random variables describing an engineering relationship so that the tolerance range can be computed. The objectives of the study are stated as follows:

1. Illustrate the development and use of the propagation of errors technique as a practically and theoretically sound method to derive an expression for the variance of a given function in terms of the random variable composing the function,
2. Compare the estimated variance from the propagation of errors analysis with the variance obtained through Monte Carlo simulation to check for substantial differences, and
3. Obtain a visible representation of the distribution for a given mathematical expression and present methodology and theory to help analyze its form.

To address the problem and apply the results obtained through simulation and propagation of error analysis, a computer algorithm has been developed to analyze several functions of different form. The program will compute percentage error, the standard deviation, and mean value for an expression using the error approximation technique. The mean and standard deviation are also obtained through the simulation routine. A histogram is plotted from which the user may view the probability distribution in a graphic form. The results of the computer analysis provide the necessary data to determine tolerance limits as well as the effect of adjusting specification limits on individual variables contained in the function.

Introduction

Statistical error analysis deals with the quantitative assessment of error involved in functions of one or several random variables. For example, systematic measurement errors may occur since the instruments used to make variable measurements cannot be constructed so as to be perfectly accurate. Thus, a certain percentage error may be associated with each variable. Usually, the main thing of interest is the distribution of the assembly or combination of variables and not so much the distribution of the characteristics of the components. It is imperative to talk in terms of distributions since they represent the most realistic way of approaching the problem.

The value of a statistical approach in dealing with this type of problem is evident as illustrated by the following example. A traditional approach for an assembly states very conservatively that if each component part is anywhere between its specified limits, the assembly characteristic will be satisfactory. For example, the "stack-up" or sum of four dimensions added together might be the assembly characteristic in question. Suppose the sum is to lie between L and U. One approach says set $L_1, \ldots, L_4$ so that if all the components are at their lower limits then $L_1 + L_2 + L_3 + L_4 = L$, and similarly $u_1 + u_2 + u_3 + u_4 = U$. This approach obviously neglects probabilities and distributions. The probability of the four components being right at their four lower limits is extremely remote. This elementary example provides a starting point for the theory of combining distributions presented in this study.

Background and Value of the Study

A multitude of work has been directed towards the determination of statistical tolerance for various mathematical relationships. Duncan (1) presents expressions with two variables for the variance of a sum or difference, of a product, and a quotient. The use of such distribution for sums and differences and for products and quotients is valid only if one has a knowledge of the distributions of the individual measurements that go to make up the particular expression. It should be noted that in Duncan's expressions, the variables are assumed to be independent of each other and to have small, measurable errors with respect to their mean values.

For engineering relationships that are too complex for general expressions, an approximation technique called propagation of errors may be used to not only calculate the overall variance, but to derive the variance expression so that the effect of individual variables may be analyzed. Development and application of this technique may be found in Denning (2), Agterberg (3), Mouradian (4), Burr (5), Kapur and Lamberson (6) and Bedient and Rainville (7). This approximation method is certainly not the only way to compute overall variance but it is relatively easy to use given that the differentiation involved with the general expression is not extremely complex. Even when lengthy theoretical distributions can be derived for an expression, the numerical techniques involved in evaluating the derived function introduce error into the computed values. From a computer programming viewpoint, the coding involved with the propagation of errors technique is simple and the input required for such a program is relatively easy even for a user with minimal background in this area. The assumptions and limitations involved with using this approximation technique are the subject of more detailed discussion included later in the study.

Given the large amount of effort devoted to obtaining the variance of a function containing random variables, much less has been said in the same literature concerning the distribution of the overall expression. Simulation and statistical goodness of fit tests are valuable tools which should be used for detailed analysis of any expression. This study incorporates traditional methodology associated with this type of work. Theoretical evidence is presented which illustrates that general statements concerning the distribution of a function composed of normally distributed, independent random variables cannot be made.

Summary

Statistical characterizations of functions of random variables provide the most realistic approach in handling such functions when they are involved in engineering design calculations. The propagation of errors approximation technique offers a relatively simple method for deriving the variance expression for a given function. The calculated variance may be used in determining the tolerance range, to help set the desired specification limits, or computing probability limits for the mean of the expression. Using the expression for overall variance, the effect of individual variable errors on the function as a whole may be determined. This study will show how to calculate the variance for some commonly used functions. Examples are presented which illustrate the use of the approximation technique in engineering design problems.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

Generally, it is very difficult to find the density function for a function of random variables. The theoretical derivation for such functions is for most cases a lengthy process involving mathematical techniques outside the scope of this study. In this chapter, the mean and variance relationships for some elementary functions are presented. Next, the propagation of errors approximation technique is described along with certain assumptions concerning its use. Finally, examples of the use of this technique are presented to illustrate its value in engineering type applications.

Statistics of Elementary Functions

In order to build upon the idea of statistical characterization of functions in general, it is important to understand the difficulties associated with first the simple cases. The following derivation focuses upon a familiar general form used in engineering work, particularly in assembly work.

Given the general expression for sums and differences

$$y = a_1x_1 + a_2x_2 + \ldots + a_kx_k \tag{2.1}$$

it is wished to determine the expressions for the mean and variance as well as to characterize the distribution of the overall function. The assumption will be made that the x_i's are independent and normally distributed. The following derivations are for the case k = 2.

The marginal density functions are found by integration:

$$f_1(x_1) = \int_{-\infty}^{\infty} f(x_1,x_2)dx_2 \ ,$$

$$f_2(x_2) = \int_{-\infty}^{\infty} f(x_1,x_2)dx_1 \ . \qquad (2.2)$$

The respective means are given by

$$\mu_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 f(x_1,x_2)dx_2dx_1 = \int_{-\infty}^{\infty} x_1 \int_{-\infty}^{\infty} f(x_1,x_2)dx_2dx_1$$

$$= \int_{-\infty}^{\infty} x_1 f_1(x_1)dx_1 = E(x_1) \ . \qquad (2.3)$$

Also,

$$\mu_2 = \int_{-\infty}^{\infty} x_2 f_2(x_2)dx_2 = E(x_2) \ . \qquad (2.4)$$

The definition of population variance is

$$\sigma_1^2 = E[(x_1 - \mu_1)^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x_1 - \mu_1)^2 f(x_1,x_2)dx_2dx_1$$

$$= \int_{-\infty}^{\infty} (x_1 - \mu_1)^2 f_1(x_1)dx_1 \qquad (2.5)$$

and similarly for x_2.

The definition of the covariance of x_1 and x_2 is

$$E[(x_1 - \mu_1)\cdot(x_2 - \mu_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1 - \mu_1)(x_2 - \mu_2)f(x_1,x_2)dx_1dx_2$$

$$= \text{covar}(x_1,x_2). \tag{2.6}$$

If x_1 and x_2 are independent then by definition

$$f(x_1,x_2) = f_1(x_1)\cdot f_2(x_2)$$

and the covariance expression becomes

$$E[(x_1 - \mu_1)\cdot(x_2 - \mu_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1 - \mu_1)(x_2 - \mu_2)f_1(x_1)f_2(x_2)dx_1dx_2$$

$$= [\int_{-\infty}^{\infty}(x_1 - \mu_1)f_1(x_1)dx_1][\int_{-\infty}^{\infty}(x_2 - \mu_2)f_2(x_2)dx_2] \tag{2.7}$$

$$= 0\cdot 0 = 0.$$

With this background, consider (2.1) with k = 2

$$\mu_y = E[a_1x_1 + a_2x_2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(a_1x_1 + a_2x_2)f(x_1,x_2)dx_1dx_2$$

$$= a_1E(x_1) + a_2E(x_2) = a_1\mu_1 + a_2\mu_2 \tag{2.8}$$

$$\sigma_y^{\,2} = E[(y - \mu_y)^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[(a_1x_1 + a_2x_2)$$

$$- (a_1\mu_1 + a_2\mu_2)]^2 f(x_1,x_2)dx_1dx_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[a_1^{\,2}(x_1 - \mu_1)^2 + 2a_1a_2(x_1 - \mu_1)(x_2 - \mu_2)$$

$$+ a_2^{\,2}(x_2 - \mu_2)^2]f_1(x_1)f_2(x_2)dx_1dx_2\ . \tag{2.9}$$

Breaking this expression up into three separate double integrals gives

$$a_1^2\sigma_1^2, \text{ zero, and } a_2^2\sigma_2^2$$

which yields for the independent case

$$\sigma_y^2 = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 \ . \tag{2.9a}$$

It can be shown that if the x_i's are normally distributed, then y is also normally distributed. In fact, for this particular expression only, the distribution of y tends towards normality even if the x_i's are not normally distributed. The historical development describing this phenomenon is discussed in depth by Adams (8). In general, it is safe to say that a linear combination of normally distributed random variables is also normally distributed.

Duncan (1) gives variance expressions for the product and quotient of two independent random variables. The statement is made that the distribution of the product or quotient will, in practical applications, be approximately normal if the distribution of the individual variables are both normal and if the division variable does not in practice become zero or take on values very close to zero. One problem associated with general type statements is the quantification of the word approximate and the bearing it has on the characterization of a particular expression.

Propagation of Error

The application of Taylor's series for error analysis of functions of several variables lends a valuable approximation technique which has been used successfully in many engineering applications. This technique, also known as propagation of errors analysis, can be used to approximate the mean and variance of a function. The theoretical development for this technique is presented as follows, first for a function of one variable and then for a function of n random variables.

The expansion of $y = f(x)$ about the point $x = \mu$ using Taylor's series up to the first three terms is

$$y = f(x) = f(\mu) + (x - \mu)f'(\mu) + \frac{(x - \mu)^2}{2!} f''(\mu) + R \qquad (2.10)$$

where R is the remainder. The expectation of this expansion is

$$\begin{aligned} E(y) &= E[f(\mu)] + E\{xf'(\mu) - \mu f'(\mu)\} + E\{\frac{1}{2} f''(\mu)(x - \mu)^2\} + E(R) \\ &= f(\mu) + \{\mu f'(\mu) - \mu f'(\mu)\} + \frac{1}{2} f''(\mu)V(x) + E(R) \\ &\approx f(\mu) + \frac{1}{2} f''(\mu)V(x) \ . \end{aligned} \qquad (2.11)$$

This is an approximation for the expected value of y since the terms making up the remainder have been ignored. The second term could also be ignored if the variance is small. To approximate the variance for y, the first two terms of Equation (2.10) will be considered.

$$V(y) \approx V[f(\mu)] + V[(x - \mu)f'(\mu)] \approx [f'(\mu)]^2 V(x) \ . \qquad (2.12)$$

Once again, since the remainder has been ignored, this is only an approximation. Thus, this procedure may not give a good approximation for a highly nonlinear function and additional terms would need to be included. The overlooking of remainder terms represents a potential hazard in relying on this method for good results.

The same approximation will now be applied to a function of n random variables of the form

$$y = f(x_1, x_2, \ldots, x_n) = f(x) \; . \tag{2.13}$$

Letting $(\mu_1, \ldots, \mu_n)$ and $(\sigma_1, \ldots, \sigma_n)$ denote the expected values and standard deviations of $x_1, \ldots, x_n$, respectively, the Taylor's series expansion results in the following approximations.

$$y = f(x_1, x_2, \ldots, x_n) = f(\mu_1, \ldots, \mu_n) + \sum_{i=1}^{m} \left. \frac{\partial f(x)}{\partial x_i} \right|_{x=\mu} (x_i - \mu)$$

$$+ \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \left. \frac{\partial^2 f(x)}{\partial x_i \partial x_j} \right|_{x=\mu} (x_i - \mu_i)(x_j - \mu_j) + R \; . \tag{2.14}$$

Taking the expectation of Equation (2.14) gives

$$E(y) = f(\mu_1, \ldots, \mu_n) + \sum_{i=1}^{n} \left. \frac{\partial f(x)}{\partial x_i} \right|_{x=\mu} E(x_i - \mu_i)$$

$$+ \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \left. \frac{\partial^2 f(x)}{\partial x_i \partial x_j} \right|_{x=\mu} E[(x_i - \mu_i)(x_j - \mu_j)] + E(R).$$

Assuming $x_1, \ldots, x_n$ to be independent random variables having correlation coefficients of zero, the zero terms may be taken out of the expression to get

$$E(y) = f(\mu_1, \ldots, \mu_n) + \frac{1}{2!} \sum_{i=1}^{n} \left. \frac{\partial^2 f(x)}{\partial x_i^2} \right|_{x=\mu} V(x_i) + E(R) \; . \tag{2.15}$$

The approximation results by dropping the remainder term to obtain

$$E(y) \approx f(\mu_1, \ldots, \mu_n) + \frac{1}{2!} \sum_{i=1}^{n} \left. \frac{\partial^2 f(x)}{\partial x_i^2} \right|_{x=\mu} V(x_i). \tag{2.16}$$

Further ignoring the second term in Equation (2.16) gives a simple but probably not as good approximation, depending on the function and the amount of error one associates with the word approximation.

$$E[f(x)] \approx f(\mu_1, \ldots, \mu_n) \; . \tag{2.17}$$

Considering only the first two terms of Equation (2.14) and taking the variance results in

$$V(y) \approx V\left[f(x)\Big|_{x=\mu}\right] + V\left[\sum_{i=1}^{n} \frac{\partial f(x)}{\partial x_i}\Big|_{x=\mu} (x_i - \mu_i)\right]$$

$$= \sum_{i=1}^{n} \left(\frac{\partial f(x)}{\partial x_i}\Big|_{x=\mu}\right)^2 V(x_i) \; . \tag{2.18}$$

A logical follow-up application of this approximation method is the inverse problem presented by Adams (8). Suppose that a certain required accuracy for a function is given in advance. This could be considered a limiting absolute error for the function. The problem is to determine the limiting absolute errors in the arguments in such a way to ensure this given accuracy in the function. For functions of more than one variable, the solution procedure must follow certain conventions as described by Adams (8) but not dealt with further in this study.

Design of an I-Beam

This problem is taken from Kapur and Lamberson (6) and illustrates the use of the propagation of errors approximation in the design of an I-beam with a certain desired reliability. The beam, shown in Figure 1, is simply supported at A and B. The weight of the beam will not be considered in this example. The beam has roller supports at B and is

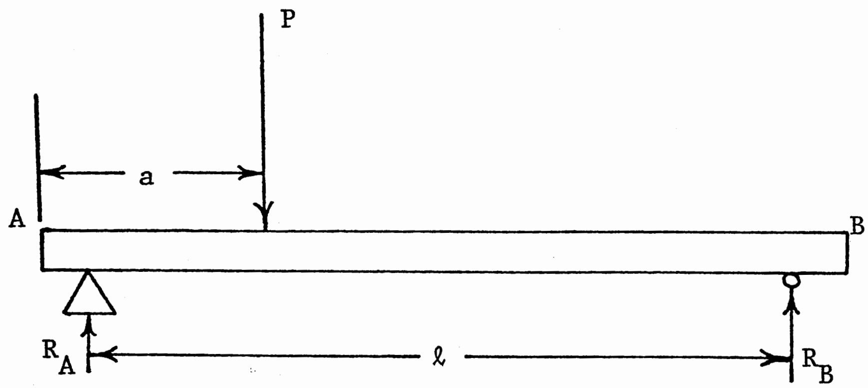

Figure 1. Simply Supported Beam

free to rotate at A and B along the longitudinal axis.

At the point of application of the load, the maximum moment M is given by

$$M = \frac{Pa(\ell - a)}{\ell} \tag{2.19}$$

where

P = applied load on the beam

ℓ = length of the beam

a = distance from endpoint of load application.

The maximum stress, occurring at the top of the upper flange or at the bottom of the lower flange of the beam is given by

$$S = \frac{Mc}{I} \tag{2.20}$$

where

S = fiber stress in psi

M = external bending moment in lb-in

c = distance from neutral axis to extreme fibers in inches

I = moment of intertia of the beam cross-section about the neutral axis in in.^{4}.

It is desired to design the beam to meet a reliability value of 0.9990. The beam is to be made of a steel I-section, the significant strength of which is given by $(\bar{\delta}, \sigma_\delta)$ = (170,000, 4,760) psi. The mean values and variabilities for the design parameters are given as follows:

$(\bar{P}, \sigma_P)$ = (6,070, 200) lb.

$\ell = 120 \pm 1/8$ in., i.e., $\bar{\ell}$ = 120 in., σ_ℓ = 1/24 in.

$a = 72 \pm 1/8$ in., i.e., $\bar{a}$ = 72 in., σ_a = 1/24 in.

Using Equation (2.19) and Taylor's series approximation, the mean and variance for the external bending moment may be obtained.

$$\bar{M} = \frac{\bar{P}\bar{a}(\bar{\ell} - \bar{a})}{\bar{\ell}} = \frac{(6070)(72)(120 - 72)}{120} = 174{,}816 \text{ lb.-in.} \tag{2.21}$$

$$\frac{\partial M}{\partial P} = \frac{\bar{a}(\bar{\ell} - \bar{a})}{\bar{a}} = \frac{72(120 - 72)}{120} = 28.8 \tag{2.22}$$

$$\frac{\partial M}{\partial \ell} = \frac{\bar{P}\bar{a}^2}{\ell^2} = \frac{6070(72)^2}{(120)^2} = 2185.2 \qquad (2.23$$

$$\frac{\partial M}{\partial a} = \bar{P} - \frac{2\bar{P}a}{\bar{\ell}} = 6070 - \frac{2(6070)72}{120} = -1214 \tag{2.24}$$

$$\sigma_M{}^2 = \frac{\partial M}{\partial P}^2 \sigma_P{}^2 + \frac{\partial M}{\partial \ell}^2 \sigma_\ell{}^2 + \frac{\partial M}{\partial a}^2 \sigma_a{}^2 = (28.8)^2(200)^2 +$$

$$(2185.2)^2(.0417)^2 + (-1214)^2(.0417)^2 = 33{,}188{,}466 \tag{2.25}$$

$$\sigma_{\bar{M}} = 5{,}761 \text{ lb.-in.}$$

For a w x 8 x 67 I-section (see Figure 2)

$$\frac{b_f}{t_f} = \frac{8.287}{.933} = 8.88 \quad \frac{d}{t_w} = \frac{9}{.575} = 15.7 \quad \frac{b_f}{d} = \frac{8.287}{9} = 0.92 \; .$$

The section modulus formula for the beam is given as

$$\frac{I}{c} = \frac{b_f d^3 - (b_f - t_w)(d - 2t_f)^3}{6d} = 0.0822\, d^3. \tag{2.26}$$

Letting $\sigma_d = 0.01\,\bar{d}$, then $(\bar{I}/\bar{c}) = 0.0822\, d^3$ and $\sigma_{(I/c)}$ may be described as

$$y = \frac{I}{c}$$

$$\frac{\partial y}{\partial d} = 3(0.0822\,\bar{d}^2) \tag{2.27}$$

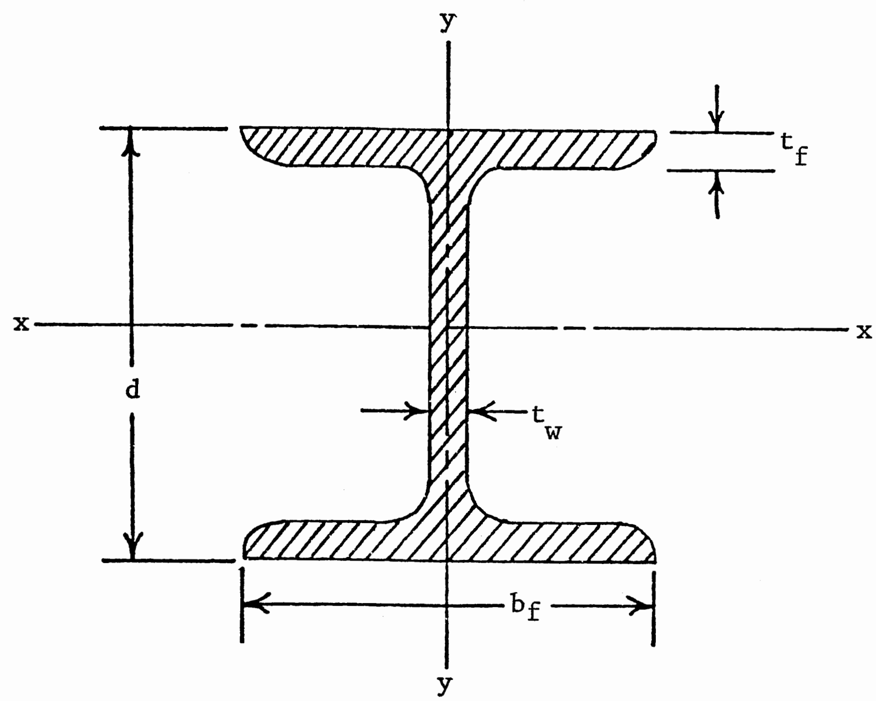

Figure 2. Cross Section of an I-Beam

$$\sigma_{(I/c)} = [(3(.0822)\bar{d}^2)^2 \sigma_d^{\,2}]^{\frac{1}{2}} = .2466\ \bar{d}^2 \sigma_d = .002466\ \bar{d}^3\ . \qquad (2.28)$$

From Equation (2.20), we have

$$\bar{S} = \frac{\bar{M}}{(\bar{I}/\bar{c})} = \frac{2{,}126{,}715}{\bar{d}^3}\ \text{psi} \qquad (2.29)$$

and

$$\sigma_S = \left[\left(\frac{1}{(\bar{I}/\bar{c})}\right)^2 \sigma_M^{\,2} + \left(\frac{-\bar{M}}{(\bar{I}/\bar{c})^2}\right)^2 \sigma^2_{(I/c)}\right]^{\frac{1}{2}} = \frac{94{,}776}{\bar{d}^3}\ . \qquad (2.30)$$

For the specified reliability of 0.999, Z is equal to -3.09, hence,

$$-3.09 = \frac{-170{,}000 + 2{,}126{,}715/\bar{d}^3}{\left[\left(\frac{94{,}776}{\bar{d}^3}\right)^2 + (4{,}760)^2\right]^{\frac{1}{2}}}\ . \qquad (2.31)$$

This simplifies to

$$\bar{d}^6 - 25.2088\ \bar{d}^3 + 154.6929 = 0\ . \qquad (2.32)$$

Solving this equation for $\bar{d}$ results in $\bar{d}$ = 2.447 in. which gives the specified reliability of 0.999.

Bearing and Shaft

A second example will illustrate how the propagation of errors analysis duplicates the results obtained using the theoretically derived variance expression for the sum of independent, random variables.

Suppose bearings and shafts are being produced for assembly. In order to insure that the shafts will all be capable of assembly at random into a bearing the following non-overlapping specification limits are set:

Inside diameter of bearing: $x_1 = .4022'' \pm .0012''$ or $.4010''$, $.4034''$.

Outside diameter of shaft: $x_2 = .4000'' \pm .0009''$ or $.3991''$, $.4009''$.

Taking the possible extreme limits, the diameterical clearance $y = x_1 - x_2$ would lie between .0001" and .0043". Suppose the production processes are capable of meeting the respective limits for x_1 and x_2 in a $\pm 3\sigma$ sense and with approximately normal distributions. Using Equation (2.8) and approximating the variance

$$\mu_y = .4022'' - .4000'' = .0022'' \tag{2.33}$$

$$\sigma_1 = .0012''/3 = .0004'' \qquad \sigma_2 = .0009''/3 = .0003'' \tag{2.34}$$

$$\frac{\partial y}{\partial x_1} = 1 \qquad \frac{\partial y}{\partial x_2} = -1 \tag{2.35}$$

$$\sigma_y^2 = (1)^2\sigma_1^2 + (-1)^2\sigma_2^2 = \sigma_1^2 + \sigma_2^2 . \tag{2.36}$$

This expression agrees with Equation (2.9a).

$$\sigma_y = [(.0004'')^2 + (.0003'')^2]^{\frac{1}{2}} = .0005''$$

In this example, y will be approximately normally distributed. Thus, about 99.7% of the diametral clearances will lie within $.0022'' \pm .0015'' = .0007''$, $.0037''$ which are obviously closer than the extreme limits. In this case, it may be desirable to run μ_1 and μ_2 closer together so as to decrease the maximum clearance.

Summary

A theoretical approach has been presented for approximating the variance and mean of a function of random variables using the Taylor's series expansion. This method, known as propagation of errors, can save considerable effort over the derivations involved with even reasonably simple functions. Examples illustrating the use of this approximation technique were given to illustrate its use and show that it can agree exactly with an expression derived from statistical definitions.

CHAPTER III

DESCRIPTION OF COMPUTER ANALYSIS

This chapter describes the computer analysis approach which was developed to analyze functions containing normally distributed, independent random variables. The program presented here was written using the time sharing option (TSO), an IBM product that allows access to the facilities of a computer center with telecommunication terminals. This form of programming is useful to the practitioner since the terminals, which transmit and receive information over telephone lines, can be used anywhere that a telephone can be installed. The user then can carry on a conversation with his computer program.

A step-by-step description of a typical data input sequence for the tolerance analysis program is presented to illustrate the type of numerical data the user needs to obtain and how it is transferred to the computer. Following the input description, an example of the output the program delivers for a particular expression is given. A FORTRAN background is not required to use the program but is necessary to augment the program so that additional functions may be added to the present list if so desired.

Data Input Sequence

The input sequence will obviously vary for different functions due to the multitude of forms these functions may assume. The user may of course program his data input sequence in any form he wishes. The discussion which follows describes and illustrates the type of input data needed for most equations and offers a clear and flexible approach for typing in the data while utilizing computer time sharing.

The following input and output example pertains to a head pressure formula given as

$$\text{HEAD} = \frac{P(2.31)}{S.G.}\left(\frac{3500}{N_T}\right)^2 \quad (3.1)$$

where

P = pressure in psi

$S.G.$ = specific gravity

N_T = test speed, rpm

The input statements are presented exactly as the computer writes them before the requested information is typed in by the user.

The very first piece of data which the computer program requests is the total number of equations to be analyzed. This total number may include different equations or the same equation containing different input values each time. For this example, it is desired to look at only one general form for one set of values. Thus, the user will type in the number one indicating his preference. The actual computer statement and reply are shown below. The question mark indicates that the program is ready to receive information.

```
 INPUT NUMBER OF EQUATIONS TO BE ANALYZED
?
1
```

The user next chooses the particular equation from those included

The user next chooses the particular equation from those included within the program that he wishes to analyze. The program being used here contains four different general expressions. These expressions are given as follows:

$$1. \quad \mu = \frac{(aX^y)\cdot(bZ^w) \times . . .}{(cR^t)\cdot(fS^v) \times . . .} \tag{3.2}$$

$$2. \quad \mu = aX + bY + cZ + . . . \tag{3.3}$$

$$3. \quad \mu = ae^{bx} \tag{3.4}$$

4. flow rate equation for a gas generator (3.5)

$$\dot{w} = .00497 \; P\sqrt{\left(\frac{P1}{P}\right)^{1.54} - \left(\frac{P1}{P}\right)^{1.77}}$$

where

$\dot{w}$ = mass flow rate, pounds mass

P = pressure in the chamber, psi

P1 = pressure at the nozzle, psi

The general form describing Equation (3.1), the pump head pressure equation, matches general form number one within the program. Thus, the input number is one. When more than one equation or set of values for one equation is to be studied, the program will return to the input sequence at this particular point following the printing of the output for the previous equation:

```
INPUT DESIRED EQUATION NUMBER
?
1
```

The user must next input the number of random variables contained within his equation. If the equation contains variables which are not random, it is probably easiest to treat the values for these variables as coefficients for the random variables:

```
INPUT NUMBER OF VARIABLES
?
3
```

The coefficient and exponent for each random variable come next in this particular input sequence. It is very important at this point to determine the order of the variables with respect to their associated input values. This same order must be kept through the next three input statements. If this order is not adhered to, the calculations will still take place but the output will be incorrect. The input is for one variable at a time, first the coefficient and then the exponent. Since this equation has three random variables, there will be a total of six question marks in the input steps indicating six values to be typed in. The order of the variables with reference to this particular expression will be the pressure value, revolutions per minute and specific gravity.

```
INPUT COEFFICIENT AND EXPONENT FOR EACH VARIABLE
?
2.31
?
1
?
3500
?
-2
?
3500
?
-1
```

Associated with each variable in this example is a measurement error. It is assumed that the percentage error for each variable is known. This error value is needed to calculate the variable standard deviation within the program. The errors should be entered as plus or minus errors and as percentage values.

```
INPUT VARIABLE PERCENTAGE ERROR
?
.419
?
.360
?
.05
```

The standard deviation spread comes next in the input sequence. Given the percentage error and the mean value to be obtained in the next step, an upper and lower limit on the range of values a variable could possibly take on may be computed. The standard deviation spread is simply the number of standard deviations the user desires to have between the two limits. Usually this spread value is six meaning practically all random values will fall within a plus or minus three standard deviation range around the mean. The normal distribution has been assumed for each variable.

```
INPUT VARIABLE STANDARD DEVIATION SPREAD
?
6
?
6
?
6
```

The nominal value for each variable is simply the mean or average value associated with each variable in the formula.

```
INPUT VARIABLE NOMINAL VALUES
?
1600
?
3510
?
.9965
```

Based upon the input data up to this point, the computer calculates the standard deviation upon the propagation of errors approach as well as the mean value of the function. Using this mean and standard deviation, the lower limit and approximate upper limit for the histogram are calculated and printed out for the user to observe. These limits are equal to the mean plus and minus four standard deviations. By viewing these limits, the user may more easily choose an appropriate interval length for the histogram.

```
APPROXIMATE BOUNDS FOR HISTOGRAM

LOWER BOUND=      3646.8625
UPPER BOUND=      3728.9341
```

The interval length is entirely up to the user but should probably result in about 10 intervals for the histogram. The interval length to use for a desired number of intervals can be determined by simple arithmetic.

```
 INPUT HISTOGRAM INTERVAL FORM
?
5
```

The number of iterations for simulation refers to the number of random formula values which the computer calculates. These values are used to calculate a simulation mean and standard deviation and are also used for the frequency table and histogram.

```
 INPUT NUMBER OF ITERATIONS
?
500
```

The user is next asked a question as to whether or not he desires to calculate probability limits. These limits give a range of values over which the mean formula value will fall a certain percentage of the time. The probability limits computed within the program are based upon the assumption that the overall distribution for the user chosen expression is approximately normal. If no limits are desired, the user types an N and the computer results begin to be printed. If limits are desired, the user types a Y. The response is another question asking the user if he wants to view all 23 possible probability limit calculations. Assuming yes answers for both of the above questions, the responses by the computer are shown below.

```
 DO YOU WISH TO CALCULATE PROBABILITY LIMITS?
 ENTER Y OR N
Y

 DO YOU WISH TO VIEW THE LIMIT CALCULATIONS?

 ENTER Y OR N
Y

( 1)  99.9%

( 2)  99.8%

( 3)  99.7%

( 4)  99.6%

( 5)  99.5%

( 6)  99.4%

( 7)  99.3%

( 8)  99.2%

( 9)  99.1%

(10)  99.0%

(11)  98.0%

(12)  97.0%

(13)  96.0%

(14)  95.0%

(15)  94.0%

(16)  93.0%

(17)  92.0%
```

(18) 91.0%

(19) 90.0%

(20) 80.0%

(21) 70.0%

(22) 60.0%

(23) 50.0%

Finally, the total number of probability limit calculations and the associated limit numbers are input values supplied by the user. The limit numbers for the calculations are simply the digits from 1 to 23.

```
 ENTER NUMBER OF LIMIT CALCULATIONS
?
3

 INPUT DESIRED LIMIT NUMBERS
?
1
?
3
?
5
```

This concludes the input sequence for the first general expression contained within the program used in this study. Output based upon the input data to the computer is now printed out at the terminal.

Program Output

The output for the computer analysis consists of five major sections. The first section contains the mean value for the function, the standard deviation and the percent error--all based upon the propagation of errors technique. The next section contains the particular probability limits which were chosen by the user if he desired to have any at all. The third section holds mean and standard deviation values obtained through simulation. The frequency table for the formula values and the histogram which graphically displays these values make up the remainder of the output. The computer output for the head pressure equation is shown in Figure 3.

Augmentation of the Program

To augment the program, the listing of which may be found in Appendix A, the programmer must consider four major objectives to be met in his coding. First, an input sequence must be established similar to the first eight steps of the previously described sequence. As mentioned before, this sequence will differ somewhat for different expressions or formulas. For the augmentation of the very program presented here, the same variable and array names corresponding to particular values of input must be used. For example, variable percentage errors are placed in the array VPE. The variable and array means have been kept the same for each expression to provide conformity and to help reduce the number of variable names within the program to avoid confusion. The user must next supply several lines of code to calculate formula random values. This may be accomplished through a simple do loop and should require about 10 lines of computer code. A counter value should be kept to hold the accumulated sum of the formula values which later are used in calculating the simulation mean and variance. Third, a sequence of about 12 program statements must be written to determine which histogram interval the formula random values fall within. These statements can be taken directly out of the program listing in Appendix A. Finally, simple calculations must be performed to solve for the simulation mean and standard deviation. About 10 statements are

```
**********************************
* PROPAGATION OF ERRORS ANALYSIS *
**********************************

FORMULA MEAN VALUE-        3687.0984

STANDARD DEVIATION-          10.2509

PERCENTAGE ERROR  -           0.8345

**********************************

      ***********************
      *  PROBABILITY LIMITS  *
      ***********************

1.  99.9%   (      3653.0306,      3721.9580)

2.  99.7%   (      3657.4292,      3718.3674)

3.  99.5%   (      3659.0706,      3716.7261)

      **********************
      * SIMULATION ANALYSIS *
      **********************
FORMULA MEAN VALUE-        3687.4319
STANDARD DEVIATION-          11.5952

        *******************
        * FREQUENCY TABLE *
        *******************

     * INTERVAL * FREQUENCY *

      3646.8625
      3651.8625            0.

      3651.8625
      3656.8625-           1.

      3656.8625
      3661.8625-           6.

      3661.8625
      3666.8625-           6.

      3666.8625
      3671.8625-          32.

      3671.8625
      3676.8625-          50.

      3676.8625
      3681.8625-          67.

      3681.8625
      3686.8625-          76.

      3686.8625
      3691.8625-          84.

      3691.8625
      3696.8625-          63.

      3696.8625
      3701.8625-          59.

      3701.8625
      3706.8625-          29.

      3706.8625
      3711.8625-          20.

      3711.8625
      3716.8625-           6.

      3716.8625
      3721.8625-           1.

      3721.8625
      3726.8625-           0.

      3726.8625
      3731.8625-           0.
```

Figure 3. Computer Output for Head Pressure Formula

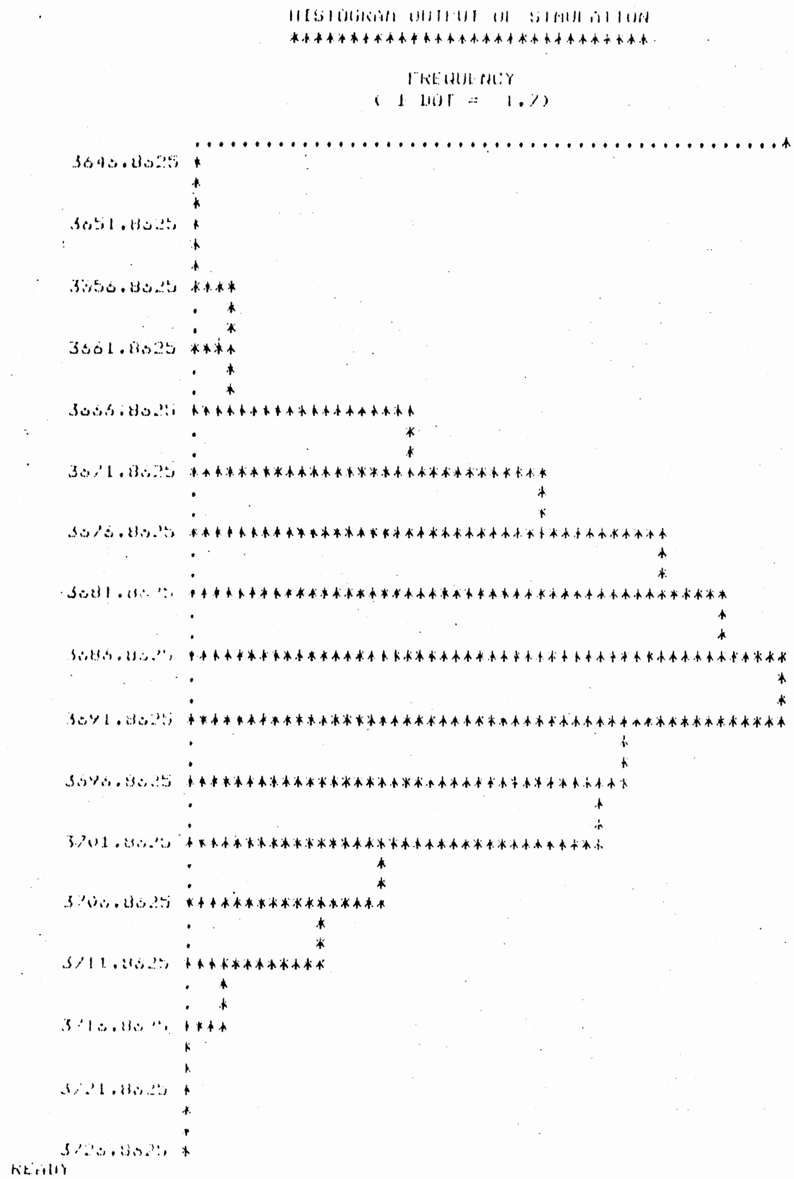

Figure 3 (Continued)

necessary to perform these calculations and write out the results.

All program output excluding the simulation calculations is handled through subroutines within the program. Additional programming for output is not necessary unless the user desires to insert more calculations for his own use. The major subroutines include LIMIT, NORMAL, RANDOM, HIST and SORT. The location in which each subroutine is called within the programming statements may be found in the program listing itself or from the gross flowchart shown in Figure 4. Detailed information concerning the subroutines may be obtained from the comment blocks within the program.

Summary

A description of the computer program used to statistically analyze different functions has been presented. Additional programming required to add a new function to the present program is minimal. Some knowledge of the FORTRAN language is required but no complicated programming techniques are necessary. A flowchart and listing of the program have been provided to illustrate and clarify the structure and logic involved.

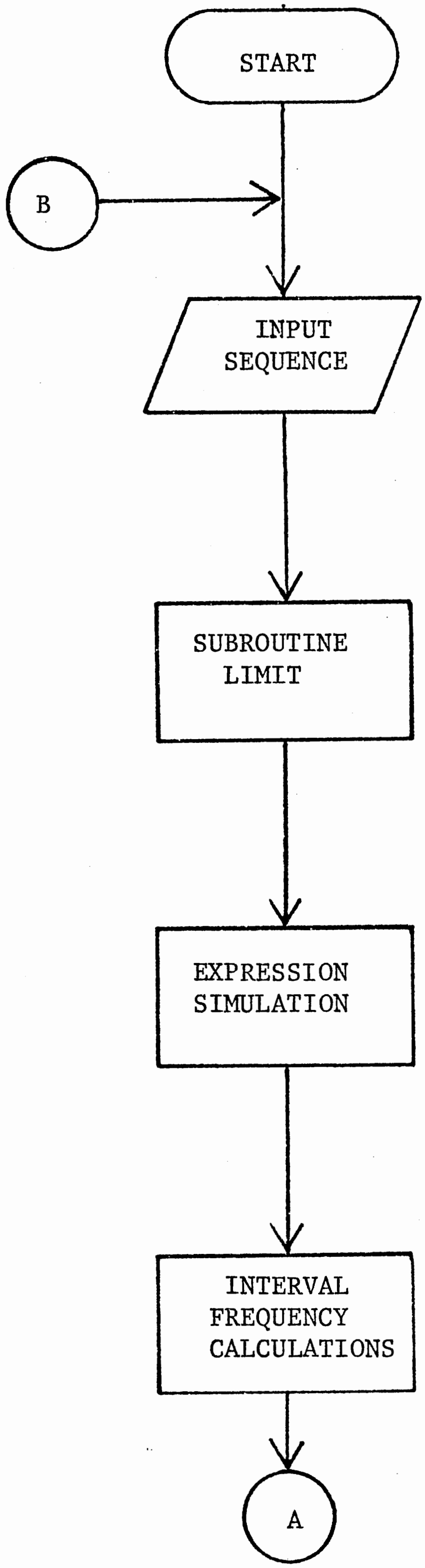

Figure 4. Gross Flowchart for Computer Program

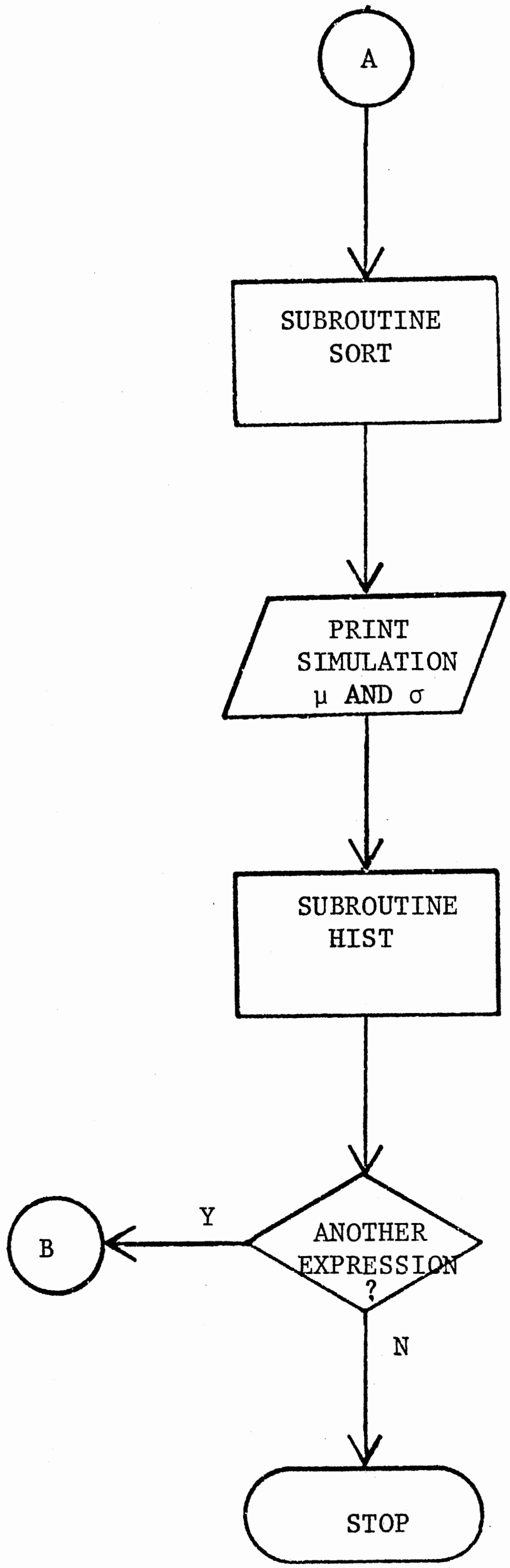

Figure 4 (Continued)

CHAPTER IV

ANALYSIS OF SEVERAL EXPRESSIONS

Introduction

To apply the theory of propagation of errors and compare the results obtained with those calculated through simulation, several functions will be analyzed using the previously described computer program. The random variables contained within each expression are assumed to be normally distributed and independent. These assumptions have been shown to be reasonable for most cases (9). The chi-square test will be conducted for each expression to test whether the frequency data obtained through simulation could readily have arisen from some normal population.

Analysis of Results

The three general expressions analyzed along with their variable values and percentage error for random variables are as follows:

$$y = a_1 x_1^{c_1} + a_2 x_2^{c_2} \tag{4.1}$$

where

$a_1 = 5.0$

$c_1 = .2$

$a_2 = 3$

$c_2 = 5$

$\bar{x}_1 = 3.5$ with $\pm$ .18% error

$\bar{x}_2 = 5.7$ with $\pm$ .15% error

$$y = a_1 e^{a_2 x} \tag{4.2}$$

where

$a_1 = 3$

$a_2 = 3$

$\bar{x} = 4.1$ with $\pm$ .567% error

$$\dot{w} = .00497\ P_c \sqrt{\left(\frac{P_n}{P_c}\right)^{1.54} - \left(\frac{P_n}{P_c}\right)^{1.77}} \tag{4.3}$$

where

$\bar{P}_c = 1000$ with $\pm$ .789% error

$\bar{P}_n = 400$ with $\pm$ .654% error

The detailed statistical output for all three expressions may be found in Appendixes B, C, and D. This output will be summarized here and several remarks concerning the results obtained and their importance are made.

Referring to Table I, the propagation of errors approximation seems to produce values for the mean and standard deviation corresponding closely with those obtained through simulation. This supports the value of the method especially when computer facilities are not available to perform simulation. The histograms for all frequency calculations seem to take on a shape similar to the bell shape of some normal type distribution.

TABLE I

COMPARISON OF MEAN AND STANDARD DEVIATION VALUES FROM SIMULATION AND PROPAGATION OF ERRORS ANALYSIS

Equation	Simulation Results		Propagation of Error Results	
	Mean	Standard Deviation	Mean	Standard Deviation
(4.1)	18056.9883	46.8278	18057.1758	45.1278
(4.2)	658922.437	16239.0664	659083.125	15321.6758
(4.3)	1.0699	.0022	1.0699	.0021

The results of the chi-square tests for all three expressions show that the sample distributions could perfectly well have come from some normal population. Thus, it would be reasonable to make probability statements for the sample averages in the form of confidence intervals while using the normal distribution to characterize the overall expression distributions.

Summary

Examples have been presented to illustrate the propagation of errors approximation as a valuable tool for computing the variance for relationships containing random variables. The results obtained from the use of this technique agree closely with those calculated based upon Monte Carlo type simulation. Additional analysis could be conducted to determine the sensitivity of the computed variance to changes in the parameters of individual distributions for variables contained within the expression. Statistical goodness of fit tests may be performed in order to approximate the observed distribution determined through the simulation output.

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of this study has been to:

1. Apply the propagation of errors technique, derived from the Taylor's series expansion of any function, to the study of statistical tolerances;
2. Develop a computer program which enables the user to use simulation and propagation of errors to determine the mean and standard deviation for a specified function as well as obtain a graphical representation of the observed frequency data; and
3. Cite possible advantages of using the approximation method as opposed to other methods but also note its drawbacks.

Based upon the results obtained and research conducted in this study, the following statements can be made:

1. The propagation of errors technique offers a good approximation for the variance and mean for a function. The errors associated with the variables contained in the function need to be small. This technique works best for functions which are not highly nonlinear due to the usual dropping of remainder terms involved in its derivation.
2. Linear functions of normally distributed random variables will be normally distributed. No general statements should be made

concerning the overall distributions of other types of functions containing normally distributed random variables.

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APPENDIXES

APPENDIX A

FORTRAN LISTING OF TOLERANCE

ANALYSIS PROGRAM

```
C*********************************************************************
C                                                                    *
C       TOLERANCE LIMIT ANALYSIS FOR ENGINEERING EXPRESSIONS         *
C*********************************************************************
C       GENERAL DESCRIPTION                                          *
C         THIS PROGRAM PERFORMS A STATISTICAL ERROR ANALYSIS ON A    *
C         USER CHOSEN EXPRESSION TO ARRIVE AT THE APPROXIMATE        *
C         VARIANCE  FOR THE OVERALL EXPRESSION. THE VARIABLES MAKING *
C         UP ANY EXPRESSION ARE ASSUMED TO BE INDEPENDENT RANDOM     *
C         VARIABLES EACH CHARACTERIZED BY THE NORMAL DISTRIBUTION. THE*
C         METHOD USED TO APPROXIMATE OVERALL VARIANCE FOR THE        *
C         EXPRESSION IS CALLED PROPAGATION OF ERRORS. THIS METHOD OF *
C         ANALYSIS IS DERIVED FROM THE TAYLOR SERIES EXPANSION OF ANY *
C         GIVEN EXPRESSION. NUMERICAL AND GRAPHICAL OUTPUT IN THE FORM*
C         OF A HISTOGRAM IS SUPPLIED BY THE PROGRAM. SPECIFIC INPUT  *
C         AND OUTPUT DATA IS AS GIVEN BELOW. THE DATA INPUT SEQUENCE *
C         MAY VARY FOR DIFFERENT EXPRESSIONS.                        *
C*********************************************************************
C       INPUT                                                        *
C         USER INPUT TO THE PROGRAM FOLLOWS 12 PROMPTING MESSAGES    *
C         GIVEN AS FOLLOWS:                                          *
C           1. INPUT THE NUMBER OF EQUATIONS TO BE ANALYZED          *
C           2. INPUT DESIRED EQUATION NUMBER                         *
C           3. INPUT NUMBER OF VARIABLES                             *
C           4. INPUT COEFFICIENT AND EXPONENT FOR EACH VARIABLE      *
C           5. INPUT VARIABLE PERCENTAGE ERROR                       *
C           6. INPUT VARIABLE STANDARD DEVIATION SPREAD              *
C           7. INPUT VARIABLE NOMINAL VALUES                         *
C           8. INPUT HISTOGRAM INTERVAL LENGTH                       *
C           9. INPUT NUMBER OF ITERATIONS FOR THE SAMPLE             *
C          10. INPUT PREFERENCE TO VIEW ALL POSSIBLE PROBABILITY     *
C              LIMIT CALCULATIONS                                    *
C          11. INPUT THE DESIRED NUMBER OF PROBABILITY LIMIT         *
C              CALCULATIONS TO PERFORM                               *
C          12. INPUT SPECIFIC PROBABILITY LIMIT CALCULATIONS         *
C*********************************************************************
C       OUTPUT                                                       *
C         OUTPUT FOR THIS PROGRAM INCLUDES:                          *
C          1. MEAN VALUE FOR THE EXPRESSION                          *
C          2. STANDARD DEVIATION FOR THE EXPRESSION                  *
C          3. PERCENT ERROR                                          *
C          4. REQUESTED PROBABILITY LIMIT   CALCULATIONS             *
C          5. FREQUENCY TABLE                                        *
C          6. HISTOGRAM                                              *
C*********************************************************************
C       GENERAL EXPRESSION FORMS CONTAINED WITHIN THIS PROGRAM ARE AS *
C       FOLLOWS:                                                     *
C                                                                    *
C           1. U=(AX**Y)*(BZ**W)*...                                 *
C                 -------------------                                *
C                 (CR**T)*(FS**V)*...                                *
C                                                                    *
C           2. U=AX + BY + CZ +...                                   *
C                                                                    *
C           3. U= AE**(BX)                                           *
C                                                                    *
C           4. FLOW RATE EQUATION FOR A GAS GENERATOR                *
C                                                                    *
C             W = .00497 * P((P1/P)**1.54 - (P1/P)**1.77)**.5        *
C                                                                    *
C             WHERE:    W= MASS FLOW RATE,POUNDS MASS                *
C                       P= PRESSURE IN THE CHAMBER,PSI               *
C                       P1=PRESSURE AT THE NOZZLE,PSI                *
C*********************************************************************
C
```

```
C
C
       DIMENSION FACT(23),XLIMIT(23),XPART(23),VRVAR(100),XN(1500),XP(150
     10),SYMBOL(50),COEFF(10),EXP(10),VPE(10),SDSP(10),VNV(10),SD(10),P(
     31500),VALUE(1500)
C
       DATA FACT/3.32,3.10,2.97,2.88,2.81,2.75,2.70,2.65,2.61,2.58,2.33,2
     1.17,2.05,1.96,1.88,1.81,1.75,1.70,1.65,1.28,1.04,0.84,0.68/
C
       DATA XLIMIT/99.9,99.8,99.7,99.6,99.5,99.4,99.3,99.2,99.1,99.0,98.0
     6,97.0,96.0,95.0,94.0,93.0,92.0,91.0,90.0,80.0,70.0,60.0,50.0/
C
       INTEGER Y,N
       DATA STAR/'*'/,BLANK/' '/,DOT/'.'/,Y/'Y'/,N/'N'/
       IX=53609
       SUM=0.0
       STDEV=0.0
       PERE=0.0
C
       WRITE(6,5)
5      FORMAT(///,2X,'INPUT NUMBER OF EQUATIONS TO BE ANALYZED')
       READ(5,*)NEQ
       DO 1000 N=1,NEQ
       XBAR1=0.0
       VALLL=0.0
       WRITE(6,10)
10     FORMAT(///,2X,'INPUT DESIRED EQUATION NUMBER')
       READ(5,*)NOEQ
       DO 6 K=1,1500
       XN(K)=0.0
       P(K)=0.0
       VALUE(K)=0.0
6      CONTINUE
       ADDIF=0.0
       ADD=0.0
       SIMEAN=0.0
       GO TO (15,265,420,565),NOEQ
C
C**********************************************************************
C     GENERAL EXPRESSION #1- U=(AX**Y)*(BZ**W)*.../(CR**T)*(FT**V)*...
C**********************************************************************
C
15     WRITE(6,20)
20     FORMAT(2X,'INPUT NUMBER OF VARIABLES')
       READ(5,*)NVAR
       WRITE(6,25)
25     FORMAT(2X,'INPUT COEFFICIENT AND EXPONENT FOR EACH VARIABLE')
       DO 30 K=1,NVAR
       READ(5,*)COEFF(K),EXP(K)
30     CONTINUE
       WRITE(6,40)
40     FORMAT(2X,'INPUT VARIABLE PERCENTAGE ERROR')
       DO 45 K=1,NVAR
       READ(5,*)VPE(K)
       VPE(K)=VPE(K)/100.0
45     CONTINUE
       WRITE(6,50)
50     FORMAT(2X,/,2X,'INPUT VARIABLE STANDARD DEVIATION SPREAD')
       DO 55 K=1,NVAR
       READ(5,*)SDSP(K)
55     CONTINUE
       WRITE(6,60)
60     FORMAT(2X,/,2X,'INPUT VARIABLE NOMINAL VALUES')
       DO 65 K=1,NVAR
       READ(5,*)VNV(K)
65     CONTINUE
```

```
C
C     CALCULATE VARIABLE STANDARD DEVIATION
C
        DO 70 K=1,NVAR
        SD(K)=((VNV(K)+VPE(K)*VNV(K))-(VNV(K)-VPE(K)*VNV(K)))/SDSF(K
       1)
   70   CONTINUE
C
C
C     CALCULATE VALUES FOR VARIANCE AND STANDARD DEVIATION
C
        FORM=1.0
        DO 90 K=1,NVAR
        X=VNV(K)**EXP(K)*COEFF(K)
        FORM=FORM*X
        C=(EXP(K)**2*SD(K)**2)/VNV(K)**2
        SUM=SUM+C
   90   CONTINUE
        VAR=FORM**2.*SUM
        STDEV=SQRT(VAR)
        PERE=((STDEV*3.)/FORM)*100.0
        XBAR1=FORM+STDEV*4.0
        VALLL=FORM-STDEV*4.0
        WRITE(6,91)
   91   FORMAT(/,2X,'APPROXIMATE BOUNDS FOR HISTOGRAM')
        WRITE(6,92)VALLL,XBAR1
   92   FORMAT(/,2X,'LOWER BOUND=',F15.4,/,2X,'UPPER BOUND=',F15.4)
        WRITE(6,93)
   93   FORMAT(/,2X,'INPUT HISTOGRAM INTERVAL LENGTH')
        READ(5,*)XLENGT
        WRITE(6,94)
   94   FORMAT(/,2X,'INPUT NUMBER OF ITERATIONS')
        READ(5,*)NITER
        NINT=IFIX((XBAR1-VALLL)/XLENGT)+1
        XYZ=VALLL
        CALL LIMIT(FORM,FACT,STDEV,XLIMIT,PERE)
C
C     BEGIN SIMULATION APPROACH
C
        FORM2=1.0
        ADD=0.0
        DO 140 K=1,NITER
        DO 100 J=1,NVAR
        CALL NORMAL(VNV,SD,VRVAR,IX,J)
        B=COEFF(J)*VRVAR(J)**EXP(J)
        FORM2=FORM2*B
  100   CONTINUE
        VALUE(K)=FORM2
        ADD=ADD+VALUE(K)
C
C      DETERMINE FREQUENCY FOR EACH INTERVAL
C
        DO 110 I=1,NINT
        IF(FORM2.LT.VALLL)GO TO 120
        VALLL=VALLL+XLENGT
        GO TO 110
  120   M=I
        GO TO 130
  110   CONTINUE
  130   CONTINUE
        XN(M)=XN(M)+1
        FORM2=1.0
        VALLL=XYZ
  140   CONTINUE
        VALLL=XYZ
C     FIND MAXIMUM FREQUENCY
```

```
      CALL SORT(NINT,P,XN)
C
      ADDIF=0.0
      SIMEAN=ADD/NITER
      DO 132 K=1,NITER
      ADDIF=ADDIF+(VALUE(K)-SIMEAN)**2
132   CONTINUE
      SIMDEV=ADDIF/NITER
      SIMDEV=SQRT(SIMDEV)
      WRITE(6,134)SIMEAN,SIMDEV
134   FORMAT(//,25X,'**********************',/,25X,'* SIMULATION ANALYS
     1IS *',/,25X,'**********************',/,20X,'FORMULA MEAN VALUE=',
     2F15.4,/,20X,'STANDARD DEVIATION=',F15.4)
      CALL HIST(XN,VALLL,XLENGT,NINT,DOT,BLANK,STAR,XYZ,P)
      DO 142 M=1,1500
      XN(M)=0.0
      P(M)=0.0
142   CONTINUE
      GO TO 1000
265   CONTINUE
C
C*************************************************************************
C     GENERAL EXPRESSION  #2- U=(AX**R)+(BY**S)+...
C*************************************************************************
      WRITE(6,270)
270   FORMAT(2X,'INPUT NUMBER OF TERMS')
      READ(5,*)NUMT
      WRITE(6,275)
275   FORMAT(2X,'INPUT COEFFICIENT AND EXPONENT FOR EACH TERM')
      DO 280 K=1,NUMT
      READ(5,*)COEFF(K),EXP(K)
280   CONTINUE
      WRITE(6,285)
285   FORMAT(2X,'INPUT VARIABLE PERCENTAGE ERROR')
      DO 290 K=1,NUMT
      READ(5,*)VPE(K)
      VPE(K)=VPE(K)/100.0
290   CONTINUE
      WRITE(6,300)
300   FORMAT(2X,'INPUT VARIABLE STANDARD DEVIATION SPREAD')
      DO 310 K=1,NUMT
      READ(5,*)SDSP(K)
310   CONTINUE
      WRITE(6,320)
320   FORMAT(2X,'INPUT VARIABLE NOMINAL VALUES')
      DO 330 K=1,NUMT
      READ(5,*)VNV(K)
330   CONTINUE
C
C   CALCULATE VARIABLE STANDARD DEVIATION
C
      DO 340 K=1,NUMT
      SD(K)=((VNV(K)+VPE(K)*VNV(K))-(VNV(K)-VPE(K)*VNV(K)))/SDSP(K)
340   CONTINUE
C
C   CALCULATE MEAN,VARIANCE AND STANDARD DEVIATION
C
      FORM=0.0
      SUM2=0.0
      DO 350 K=1,NUMT
      C=COEFF(K)*VNV(K)**EXP(K)
      FORM=FORM+C
      D=(COEFF(K)*EXP(K)*VNV(K)**(EXP(K)-1.)*SD(K))**2.
      SUM2=SUM2+D
350   CONTINUE
      STDEV=SQRT(SUM2)
```

```
      PERE=((STDEV*3.)/FORM)*100.0
      XBAR1=FORM+STDEV*4.0
      VALLL=FORM-STDEV*4.0
      WRITE(6,355)
355   FORMAT(/,2X,'APPROXIMATE BOUNDS FOR HISTOGRAM')
      WRITE(6,360)VALLL,XBAR1
360   FORMAT(/,2X,'LOWER BOUND=',F15.4,/,2X,'UPPER BOUND=',F15.4)
      WRITE(6,365)
365   FORMAT(/,2X,'INPUT HISTOGRAM INTERVAL LENGTH')
      READ(5,*)XLENGT
      WRITE(6,370)
370   FORMAT(/,2X,'INPUT NUMBER OF ITERATIONS')
      READ(5,*)NITER
      NINT=IFIX((XBAR1-VALLL)/XLENGT)+1
      XYZ=VALLL
      CALL LIMIT(FORM,FACT,STDEV,XLIMIT,PERE)
C  BEGIN SIMULATION
      ADD=0.0
      FORM2=0.0
      DO 390 K=1,NITER
      DO 375 J=1,NUMT
      CALL NORMAL(VNV,SD,VRVAR,IX,J)
      B=COEFF(J)*VRVAR(J)**EXP(J)
      FORM2=FORM2+B
375   CONTINUE
      VALUE(K)=FORM2
      ADD=ADD+VALUE(K)
      DO 385 I=1,NINT
      IF(FORM2.LT.VALLL)GO TO 380
      VALLL=VALLL+XLENGT
      GO TO 385
380   M=I
      GO TO 387
385   CONTINUE
387   CONTINUE
      XN(M)=XN(M)+1
      FORM2=0.0
      VALLL=XYZ
390   CONTINUE
      VALLL=XYZ
C   SORT THE EXPRESSION VALUES
      CALL SORT(NINT,P,XN)
C
      ADDIF=0.0
      SIMEAN=ADD/NITER
      DO 395 K=1,NITER
      ADDIF=ADDIF+(VALUE(K)-SIMEAN)**2
395   CONTINUE
      SIMDEV=ADDIF/NITER
      SIMDEV=SQRT(SIMDEV)
      WRITE(6,400)SIMEAN,SIMDEV
400   FORMAT(//,25X,'************************',/,25X,'* SIMULATION ANALYS
     1IS *',/,25X,'************************',/,20X,'FORMULA MEAN VALUE=',
     2F15.4,/,20X,'STANDARD DEVIATION=',F15.4)
      CALL HIST(XN,VALLL,XLENGT,NINT,DOT,BLANK,STAR,XYZ,P)
      DO 410 M=1,1500
      P(M)=0.0
      XN(M)=0.0
410   CONTINUE
      GO TO 1000
420   CONTINUE
C
C**********************************************************************
C   GENERAL EXPRESSION #3- U= AE**BX
C**********************************************************************
C
```

```
      WRITE(6,430)
  430 FORMAT(2X,'INPUT VALUE OF FORMULA COEFFICIENT')
      READ(5,*)COEF
      WRITE(6,440)
  440 FORMAT(2X,'INPUT COEFFICIENT VALUE FOR THE EXPONENT')
      READ(5,*)COEFE
      WRITE(6,450)
  450 FORMAT(2X,'INPUT VARIABLE MEAN VALUE')
      READ(5,*)VNV(1)
      WRITE(6,460)
  460 FORMAT(2X,'INPUT VARIABLE PERCENTAGE ERROR')
      READ(5,*)VAPE
      WRITE(6,470)
  470 FORMAT(2X,'INPUT DESIRED STANDARD DEVIATION SPREAD')
      READ(5,*)SDS
      VAPE=VAPE/100.0
C
C   CALCULATE STANDARD DEVIATION
      SD(1)=((VNV(1)+VNV(1)*VAPE)-(VNV(1)-VNV(1)*VAPE))/SDS
C
C   CALCULATE MEAN AND STANDARD DEVIATION FOR THE EXPRESSION
C
      FORM=COEF*2.71828**(COEFE*VNV(1))
      STDEV=(COEF**2*COEFE**2*SD(1)**2*2.71828**(2*COEFE*VNV(1)))**.5
      PERE=((STDEV*3.)/FORM)*100.0
      XBAR1=FORM+STDEV*4.0
      VALLL=FORM-STDEV*4.0
      WRITE(6,480)
  480 FORMAT(/,2X,'APPROXIMATE BOUNDS FOR HISTOGRAM')
      WRITE(6,490)VALLL,XBAR1
  490 FORMAT(/,2X,'LOWER BOUND=',F15.4,/,2X,'UPPER BOUND=',F15.4)
      WRITE(6,500)
  500 FORMAT(/,2X,'INPUT HISTOGRAM INTERVAL LENGTH')
      READ(5,*)XLENGT
      WRITE(6,510)
  510 FORMAT(/,2X,'INPUT NUMBER OF ITERATIONS FOR SIMULATION')
      READ(5,*)NITER
      NINT=IFIX((XBAR1-VALLL)/XLENGT)+1
      XYZ=VALLL
      CALL LIMIT(FORM,FACT,STDEV,XLIMIT,PERE)
C
C  BEGIN SIMULATION
C
      FORM2=0.0
      J=1
      DO 535 K=1,NITER
      CALL NORMAL(VNV,SD,VRVAR,IX,J)
      FORM2=COEF*2.71828**(COEFE*VRVAR(1))
      VALUE(K)=FORM2
      ADD=ADD+VALUE(K)
      DO 530 I=1,NINT
      IF(FORM2.LT.VALLL)GO TO 520
      VALLL=VALLL+XLENGT
      GO TO 530
  520 M=I
      GO TO 532
  530 CONTINUE
  532 CONTINUE
      XN(M)=XN(M)+1
      FORM2=0.0
      VALLL=XYZ
  535 CONTINUE
      VALLL=XYZ
C
C   SORT EXPRESSION VALUES
      CALL SORT(NINT,P,XN)
```

```
C
       ADDIF=0.0
       SIMEAN=ADD/NITER
       DO 540 K=1,NITER
       ADDIF=ADDIF+(VALUE(K)-SIMEAN)**2
 540   CONTINUE
       SIMDEV=(ADDIF/NITER)**.5000
       SIMEAN=ADD/NITER
       WRITE(6,550)SIMEAN,SIMDEV
 550   FORMAT(//,25X,'************************',/,25X,'* SIMULATION ANALYS
      1IS *',/,25X,'************************',/,20X,'FORMULA MEAN VALUE=',
      2F15.4,/,20X,'STANDARD DEVIATION=',F15.4)
       CALL HIST(XN,VALLL,XLENGT,NINT,DOT,BLANK,STAR,XYZ,P)
       DO 560 M=1,1500
       P(M)=0.0
       XN(M)=0.0
 560   CONTINUE
       GO TO 1000
 565   CONTINUE
C*************************************************************************
C      GENERAL EXPRESSION #4- MASS FLOW RATE EQUATION                    *
C*************************************************************************
       WRITE(6,570)
 570   FORMAT(2X,'INPUT MEAN PRESSURE VALUES-CHAMBER AND NOZZLE')
       DO 580 K=1,2
       READ(5,*)VNV(K)
 580   CONTINUE
       WRITE(6,590)
 590   FORMAT(2X,'INPUT PERCENTAGE ERROR-CHAMBER AND NOZZLE PRESSURES')
       DO 600 K=1,2
       READ(5,*)VPE(K)
       VPE(K)=VPE(K)/100.0
 600   CONTINUE
       WRITE(6,610)
 610   FORMAT(2X,'INPUT PRESSURE STANDARD DEVIATION SPREADS')
       DO 620 K=1,2
       READ(5,*)SDSP(K)
 620   CONTINUE
C
C    CALCULATE PRESSURE STANDARD DEVIATIONS
       DO 630 K=1,2
       SD(K)=((VNV(K)+VPE(K)*VNV(K))-(VNV(K)-VPE(K)*VNV(K)))/SDSP(K)
 630   CONTINUE
C
       FORM=.00497*VNV(1)*((VNV(2)/VNV(1))**1.54-(VNV(2)/VNV(1))**1.77)**
      1.5
C
C    CALCULATE DERIVATIVE VALUES
       TN1=.46*(VNV(2)/VNV(1))**1.54-.23*(VNV(2)/VNV(1))**1.77
       TD1=2*((VNV(2)/VNV(1))**1.54-(VNV(2)/VNV(1))**1.77)**.5
       T1TOT=.00497*(TN1/TD1)
       TN2=1.54*(VNV(2)/VNV(1))**.54-1.77*(VNV(2)/VNV(1))**.77
       TD2=TD1
       T2TOT=.00497*(TN2/TD2)
       STDEV=T1TOT**2*SD(1)**2+T2TOT**2*SD(2)**2
       PERE=((STDEV*3.)/FORM)*100.0
       STDEV=SQRT(STDEV)
       XBAR1=FORM+STDEV*4.0
       VALLL=FORM-STDEV*4.0
       WRITE(6,640)VALLL,XBAR1
 640   FORMAT(/,2X,'APPROXIMATE BOUNDS FOR HISTOGRAM',/,2X,'LOWER BOUND='
      1,F15.4,/,2X,'UPPER BOUND=',F15.4)
       WRITE(6,650)
 650   FORMAT(/,2X,'INPUT HISTOGRAM INTERVAL LENGTH')
       READ(5,*)XLENGT
       WRITE(6,660)
```

```
660   FORMAT(/,2X,'INPUT NUMBER OF ITERATIONS')
      READ(5,*)NITER
      NINT=IFIX((XBAR1-VALLL)/XLENGT)+1
      XYZ=VALLL
      CALL LIMIT(FORM,FACT,STDEV,XLIMIT,PERE)
C
C  BEGIN SIMULATION
      ADD=0.0
      DO 710 K=1,NITER
      DO 670 J=1,2
      CALL NORMAL(VNV,SD,VRVAR,IX,J)
670   CONTINUE
      VALUE(K)=.00497*VRVAR(1)*((VRVAR(2)/VRVAR(1))**1.54-(VRVAR(2)/VRVA
     1R(1))**1.77)**.5
      ADD=ADD+VALUE(K)
C   DETERMINE INTERVAL FREQUENCY
      DO 690 I=1,NINT
      IF(VALUE(K).LT.VALLL)GO TO 680
      VALLL=VALLL+XLENGT
      GO TO 690
680   M=I
      GO TO 700
690   CONTINUE
700   CONTINUE
      XN(M)=XN(M)+1
      VALLL=XYZ
710   CONTINUE
      VALLL=XYZ
      ADDIF=0.0
      SIMEAN=ADD/NITER
      DO 720 K=1,NITER
      ADDIF=ADDIF+(VALUE(K)-SIMEAN)**2
720   CONTINUE
      SIMDEV=ADDIF/NITER
      SIMDEV=SQRT(SIMDEV)
      WRITE(6,730)SIMEAN,SIMDEV
730   FORMAT(//,25X,'**********************',/,25X,'* SIMULATION ANALYS
     1IS *',/,25X,'**********************',/,20X,'FORMULA MEAN VALUE=',
     2F15.4,/,20X,'STANDARD DEVIATION=',F15.4)
C
      CALL SORT(NINT,P,XN)
      CONTINUE
      CALL HIST(XN,VALLL,XLENGT,NINT,DOT,BLANK,STAR,XYZ,P)
1000  CONTINUE
      STOP
      END
C*********************************************************************
C                                                                    *
C     SUBROUTINE LIMIT                                               *
C*********************************************************************
C     PURPOSE                                                        *
C        COMPUTES AND PRINTS PROBABILITY LIMITS FOR AN EXPRESSION    *
C        BASED ON THE NORMAL DISTRIBUTION. THE USER CHOOSES THE      *
C        PARTICULAR PERCENTAGE PROBABILITY LIMITS   FROM A LIST OF 23*
C        POSSIBLE CALCULATIONS. MEAN VALUE,STANDARD DEVIATION,AND    *
C        PERCENTAGE ERROR FOR THE EXPRESSION ARE ALSO PRINTED.       *
C     USAGE                                                          *
C        CALL LIMIT(FORM,FACT,STDEV,XLIMIT,PERE)                     *
C     DESCRIPTION OF PARAMETERS                                      *
C        FORM-MEAN VALUE FOR EXPRESSION                              *
C        FACT-ARRAY HOLDING Z VALUES INTERPOLATED FROM A Z TABLE     *
C        STDEV-EXPRESSION STANDARD DEVIATION                         *
C        XLIMIT-ARRAY HOLDING POSSIBLE PROBABILITY LIMIT   VALUES    *
C        PERE-EXPRESSION PERCENTAGE ERROR                            *
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  *
C        NONE                                                        *
```

```
C*************************************************************************
      SUBROUTINE LIMIT(FORM,FACT,STDEV,XLIMIT,PERE)
      DIMENSION XU(23),XL(23),XLIMIT(23),FACT(23),XPART(23)
      DATA Y/'Y'/,N/'N'/
      WRITE(6,148)
148   FORMAT(/,2X,'DO YOU WISH TO CALCULATE PROBABILITY LIMITS?',/,2X,'E
     1NTER Y OR N')
      READ(5,162)IANSW
162   FORMAT(A1)
      IF(IANSW.EQ.N)GO TO 152
      WRITE(6,143)
143   FORMAT(/,2X,'DO YOU WISH TO VIEW THE LIMIT CALCULATIONS?')
      WRITE(6,139)
139   FORMAT(/,2X,'ENTER Y OR N')
      READ(5,146)IREPLY
146   FORMAT(A1)
      IF(IREPLY.EQ.N)GO TO 144
      DO 141 K=1,23
      WRITE(6,135)K,XLIMIT(K)
141   CONTINUE
135   FORMAT(/,2X,'(',I2,')',2X,F4.1,'%')
144   WRITE(6,145)
145   FORMAT(2X,/,2X,'ENTER NUMBER OF LIMIT CALCULATIONS')
      READ(5,*)NLC
C
C   INPUT SPECIFIC LIMITS DESIRED
      WRITE(6,147)
147   FORMAT(//,2X,'INPUT DESIRED LIMIT NUMBERS')
      DO 150 K=1,NLC
      READ(5,*)XPART(K)
150   CONTINUE
152   WRITE(6,155)FORM,STDEV,PERE
155   FORMAT(////,20X,'************************************',/,20X,'* PROP
     1AGATION OF ERRORS ANALYSIS *',/,20X,'*****************************
     2*****',//,20X,'FORMULA MEAN VALUE=',F15.4,//,20X,'STANDARD DEVIATI
     3ON=',F15.4,//,20X,'PERCENTAGE ERROR  =',F15.4,//,20X,'***********
     4**********************')
C
      IF(IANSW.EQ.N)GO TO 178
C   COMPUTE AND WRITE LIMITS
      WRITE(6,179)
179   FORMAT(//,25X,'************************',/,25X,'*  PROBABILITY LIM
     1ITS  *',/,25X,'************************')
      DO 175 K=1,NLC
      X=XPART(K)
      XU(K)=FORM+FACT(X)*STDEV
      XL(K)=FORM-FACT(X)*STDEV
      WRITE(6,172)K,XLIMIT(X),XL(K),XU(K)
172   FORMAT(/,14X,I2,'.',2X,F4.1,'%',3X,'(',F15.4,',',F15.4,')')
175   CONTINUE
C
178   RETURN
      END
C
C*************************************************************************
C                                                                        *
C      SUBROUTINE NORMAL                                                 *
C*************************************************************************
C        PURPOSE                                                         *
C           COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN   *
C           MEAN AND STANDARD DEVIATION                                  *
C        USAGE                                                           *
C           CALL NORMAL(VNV,SD,VRVAR,IX,J)                               *
C        DESCRIPTION OF PARAMETERS                                       *
C           VNV-ARRAY HOLDING VARIABLE MEAN VALUES                       *
C           SD-ARRAY HOLDING VARIABLE STANDARD DEVIATIONS                *
```

```
C         VRVAR-THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE          *
C         IX-IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR LESS      *
C           DIGITS ON THE FIRST ENTRY TO NORMAL.THEREAFTER IT WILL        *
C           CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM NUMBER         *
C           GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT ENTRY TO      *
C           THE SUBROUTINE.                                               *
C          J-COUNTER VALUE FOR A PARTICULAR VARIABLE IN THE               *
C            EXPRESSION                                                   *
C       METHOD                                                            *
C         USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM         *
C         NUMBERS BY CENTRAL LIMIT THEOREM.THE RESULT IS THEN ADJUSTED*
C         TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.                 *
C       SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED                      *
C         RANDUM                                                          *
C                                                                         *
C*************************************************************************
C
C
      SUBROUTINE NORMAL(VNV,SD,VRVAR,IX,J)
      DIMENSION VRVAR(100),VNV(10),SD(10)
      A=0.0
      DO 180 I=1,12
      CALL RANDUM(IX,IY,RN)
      A=A+RN
180   CONTINUE
      VRVAR(J)=VNV(J)+(A-6.)*SD(J)
      RETURN
      END
C
C*************************************************************************
C                                                                         *
C       SUBROUTINE RANDUM                                                 *
C*************************************************************************
C       PURPOSE                                                           *
C        COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN 0 *
C        AND 1.0 AND RANDOM INTEGERS BETWEEN 0 AND 2**31. EACH ENTRY      *
C        USES AS INPUT AN INTEGER RANDOM NUMBER AND PRODUCES A NEW        *
C        INTEGER AND REAL RANDOM NUMBER                                   *
C       USAGE                                                             *
C         CALL RANDUM(IX,IY,RN)
C       DESCRIPTION OF PARAMETERS                                         *
C         IX-FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER        *
C            NUMBER WITH NINE OR LESS DIGITS.AFTER THE FIRST ENTRY,       *
C            IX SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS       *
C            SUBROUTINE                                                   *
C         IY-A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT      *
C            ENTRY TO THIS SUBROUTINE.THE RANGE OF THIS NUMBER IS         *
C            BETWEEN 0 AND 2**31.                                         *
C         RN-THE RESULTING UNIFORMLY DISTRIBUTED,FLOATING POINT,          *
C            RANDOM NUMBER IN  THE RANGE OF 0 TO 1.0.                     *
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                     *
C        NONE                                                             *
C*************************************************************************
C
      SUBROUTINE RANDUM(IX,IY,RN)
      IY=IX*65539
      IF(IY)185,190,190
185   IY=IY+2147483647+1
190   RN=IY
      RN=RN*.4656613E-9
      IX=IY
      RETURN
      END
C
C*************************************************************************
C                                                                         *
```

```
C      SUBROUTINE HIST                                                   *
C***********************************************************************
C      PURPOSE                                                           *
C         SCALES AND PLOTS A HISTOGRAM FOR ARRAY DATA PASSED FROM THE   *
C         MAIN PROGRAM. THE FREQUENCY SCALE IS DETERMINED BY THE        *
C         MAXIMUM ARRAY VALUE. SIZE OF THE HISTOGRAM PRINTOUT MAY BE    *
C         ADJUSTED BY CHANGING THE SIZE OF THE ARRAY 'SYMBOL'. A        *
C         FREQUENCY TABLE IS ALSO PRINTED.                              *
C      USAGE                                                             *
C         CALL HIST(XN,VALLL,XLENGT,NINT,DOT,BLANK,STAR,P)              *
C      DESCRIPTION OF PARAMETERS                                         *
C         XN-ARRAY HOLDING EXPRESSION VALUES FROM SIMULATION            *
C         VALLL-VALUE OF LOWER LIMIT ON HISTOGRAM                       *
C         XLENGT-LENGTH OF AN INTERVAL ON THE HISTOGRAM                 *
C         NINT-NUMBER OF INTERVALS                                      *
C         DOT,BLANK,STAR-PLOTTING SYMBOLS                               *
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                     *
C         NONE                                                          *
C***********************************************************************
       SUBROUTINE HIST(XN,VALLL,XLENGT,NINT,DOT,BLANK,STAR,XYZ,P)
       DIMENSION XN(1500),P(1500),SYMBOL(50)
       WRITE(6,192)
192    FORMAT(///,27X,'*******************',/,27X,'* FREQUENCY TABLE *',/
      1,27X,'*******************')
       WRITE(6,195)
195    FORMAT(//,24X,'* INTERVAL * FREQUENCY *')
       DO 205 K=1,NINT
       F=VALLL+XLENGT
       K2=K+1
       WRITE(6,200)VALLL,F,XN(K2)
200    FORMAT(/,19X,F15.4,/,19X,F15.4,'-',2X,F10.0)
       VALLL=VALLL+XLENGT
205    CONTINUE
C     SCALE AND PLOT HISTOGRAM
       WRITE(6,230)
230    FORMAT(/////,23X,'HISTOGRAM OUTPUT OF SIMULATION',/,23X,'*********
      1*********************')
       XXX=P(NINT)/49
       WRITE(6,246)XXX
246    FORMAT(2X,/,33X,'FREQUENCY',/,30X,'( 1 DOT =',F5.1,')')
       DO 235 J=1,49
235    SYMBOL(J)=DOT
       SYMBOL(50)=STAR
       WRITE(6,240)SYMBOL
240    FORMAT(/,15X,50A1)
C
       VALLL=XYZ
       DO 247 K=1,50
247    SYMBOL(K)=BLANK
       DO 260 K=2,NINT
       IF(K.EQ.1)GO TO 258
       IF(XN(K).LT.XN(K-1))GO TO 253
258    ISTAR=IFIX(49*XN(K)/P(NINT))+1
       DO 248 I=1,ISTAR
       SYMBOL(I)=STAR
248    CONTINUE
       GO TO 257
253    DO 255 I=1,ISTAR
255    SYMBOL(I)=STAR
257    WRITE(6,249)VALLL,SYMBOL
249    FORMAT(2X,F12.4,1X,50A1)
       DO 251 J=1,50
251    SYMBOL(J)=BLANK
       IF(K.EQ.1)GO TO 254
       IF(XN(K).GT.XN(K-1))GO TO 254
       ISTAR=IFIX(49*XN(K)/P(NINT))+1
254    SYMBOL(1)=DOT
```

```
      SYMBOL(ISTAR)=STAR
      WRITE(6,252)SYMBOL,SYMBOL
252   FORMAT(15X,50A1,/,15X,50A1)
      VALLL=VALLL+XLENGT
260   CONTINUE
      WRITE(6,256)VALLL,SYMBOL
256   FORMAT(2X,F12.4,1X,50A1)
      RETURN
      END
C*********************************************************************
C                                                                    *
C     SUBROUTINE SORT                                                *
C*********************************************************************
C     PURPOSE                                                        *
C        ARRANGES UP TO 1500 REAL NUMBERS INTO ASCENDING ORDER       *
C     USAGE                                                          *
C        CALL SORT(NINT,P,XN)                                        *
C     DESCRIPTION OF ARGUMENTS                                       *
C        NINT-NUMBER OF INTERVALS ON THE HISTOGRAM                   *
C@       P-SORTED ARRAY                                              *
C        XN-ARRAY HOLDING VALUES TO BE SORTED                        *
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  *
C*********************************************************************
C
      SUBROUTINE SORT(NINT,P,XN)
      DIMENSION P(1500),XN(1500)
      DO 750 K=1,NINT
      P(K)=XN(K)
750   CONTINUE
      NPI=NINT-1
      DO 790 I=1,NPI
      I1=I+1
      DO 790 J=I1,NINT
      IF(P(J).GE.P(I))GO TO 790
      KSAVE=P(I)
      P(I)=P(J)
      P(J)=KSAVE
790   CONTINUE
      RETURN
      END
END OF DATA
```

APPENDIX B

COMPUTER OUTPUT: CHI-SQUARE TEST

FOR GENERAL EXPRESSION 2

*** PROPAGATION OF ERRORS ANALYSIS ***

FORMULA MEAN VALUE 18057.1250

STANDARD DEVIATION 45.1250

PERCENTAGE ERROR = 0.2497

*** PROBABILITY LIMITS ***

1. 99.0% (17917.2773, 18197.0703)
2. 99.5% (17930.3633, 18183.9844)

*** SIMULATION ANALYSIS ***

FORMULA MEAN VALUE 18056.9063
STANDARD DEVIATION= 46.8270

*** FREQUENCY TABLE ***

* INTERVAL	* FREQUENCY *
17876.6641 17901.6641-	1.
17901.6641 17926.6641-	0.
17926.6641 17951.6641	7.
17951.6641 17976.6641	23.
17976.6641 18001.6641-	59.
18001.6641 18026.6641-	99.
18026.6641 18051.6641-	165.
18051.6641 18076.6641-	149.
18076.6641 18101.6641-	113.
18101.6641 18126.6641-	76.
18126.6641 18151.6641-	40.
18151.6641 18176.6641-	14.
18176.6641 18201.6641	4.
18201.6641 18226.6641-	0.
18226.6641 18251.6641-	0.

```
                         HISTOGRAM OUTPUT OF SIMULATION
                         ******************************

                                   FREQUENCY
                                 ( 1 DOT =  3.4)

              .................................................*
   17876.6641 *
              *
              *
   17901.6641 *
              *
              *
   17926.6641 ***
              . *
              . *
   17951.6641 *******
              .     *
              .     *
   17976.6641 ******************
              .                *
              .                *
   18001.6641 ******************************
              .                            *
              .                            *
   18026.6641 **************************************************
              .                                                *
              .                                                *
   18051.6641 **************************************************
              .                                           *
              .                                           *
   18076.6641 *********************************************
              .                                *
              .                                *
   18101.6641 **********************************
              .                      *
              .                      *
   18126.6641 ***********************
              .          *
              .          *
   18151.6641 ************
              .   *
              .   *
   18176.6641 *****
              .*
              .*
   18201.6641 **
              *
              *
   18226.6641 *
READY
```

TABLE II

OBSERVED DISTRIBUTION OF 750 RANDOM VALUES FOR THE EXPRESSION $y = ax^R + by^S$
CLASS FREQUENCIES CALCULATED UNDER HYPOTHESIS OF NORMALITY,
AND GOODNESS OF FIT TESTED BY CHI-SQUARE

(1) Mid Value x	(2) Observed Freq., f_i	(3) Class Bounds, x	(4) Standard Var. Z	(5) Area Below Z in (4)	(6) Area Between	(7) Calcul. Freq., F_i	(8) Contr. to x^2
17889.16	1				.0005	.375	1.04
		17901.66	-3.317	.0005			
17914.16	0				.0022	1.65	1.65
		17926.66	-2.783	.0027			
17939.16	7				.0095	7.13	.0023
		17951.66	-2.249	.0122			
17964.16	23				.0305	22.88	.0066
		17976.66	-1.715	.0427			
17989.16	59				.0763	57.2	.057
		18001.66	-1.181	.1190			
18014.16	99				.1388	104.1	.250
		18026.66	-0.648	.2578			
18039.16	165				.1984	148.8	1.76
		18051.66	-0.114	.4562			
18064.16	149				.2066	155.0	.232
		18076.66	0.420	.6628			
18089.16	113				.1661	124.6	1.08
		18101.66	0.954	.8289			
18114.16	76				.1030	77.3	.022
		18126.66	1.488	.9319			
18139.16	40				.0464	34.8	.78
		18151.66	2.022	.9783			
18164.16	14				.0165	12.4	.21
		18176.66	2.556	.9948			
18189.16	4				.0052	3.9	.0026
							7.087

$v = 13 - 1 - 2 = 10$. $\alpha = .01$. $x^2_{10,.99} = 23.209$. Fit is satisfactory.

APPENDIX C

COMPUTER OUTPUT: CHI-SQUARE TEST

FOR GENERAL EXPRESSION 3

```
**********************************
* PROPAGATION OF ERRORS ANALYSIS *
**********************************

FORMULA MEAN VALUE =     659083.125

STANDARD DEVIATION=      15321.6750

PERCENTAGE ERROR  =          6.9741

**********************************

     ************************
     *  PROBABILITY LIMITS  *
     ************************

1.  99.9%   (    608215.125 ,    709951.062 )

2.  99.7%   (    613577.687 ,    704500.500 )

3.  99.5%   (    616029.187 ,    702137.000 )

     ***********************
     * SIMULATION ANALYSIS *
     ***********************
FORMULA MEAN VALUE       658927.437
STANDARD DEVIATION        16239.0664


       ********************
       * FREQUENCY TABLE  *
       ********************

  * INTERVAL  * FREQUENCY *

   597298.375
   609298.375 -          1.

   609298.375
   621298.375 -          3.

   621298.375
   633298.375 -         52.

   633298.375
   645298.375 -        152.

   645298.375
   657298.375 -        270.

   657298.375
   669298.375 -        268.

   669298.375
   681298.375 -        169.

   681298.375
   693298.375 -         69.

   693298.375
   705298.375 -         15.

   705298.375
   717298.375 -          1.

   717298.375
   729298.375 -          0.
```

HISTOGRAM OUTPUT OF SIMULATION
****

```
                              FREQUENCY
                            ( 1 DOT =  5.5)

            ..................................................*
597796.375  *
            *
            *
609796.375  *
            *
            *
621796.375  **********
            .        *
            .        *
633796.375  ****************************
            .                          *
            .                          *
645796.375  ***************************************************
            .                                                 *
            .                                                 *
657796.375  ***************************************************
            .                                                *
            .                                                *
669796.375  **************************************************
            .                             *
            .                             *
681796.375  *******************************
            .           *
            .           *
693796.375  *************
            . *
            . *
705796.375  ***
            *
            *
717796.375  *
```

TABLE III

OBSERVED DISTRIBUTION OF 1000 RANDOM VALUES OF THE EXPRESSION ae^{bx}
CLASS FREQUENCIES CALCULATED UNDER HYPOTHESIS OF NORMALITY,
AND GOODNESS OF FIT TESTED BY CHI-SQUARE

(1) Mid Value x	(2) Observed Freq., f_i	(3) Class Bounds, x	(4) Standard Var. Z	(5) Area Below Z in (4)	(6) Area Between	(7) Calcul. Freq., F_i	(8) Contr. to x^2
603,796.375	1				.0013	1.3	.069
		609,796.375	-3.025	.0013			
615,796.375	3				.0097	9.7	4.63
		621,796.375	-2.286	.0110			
627,796.375	52				.0496	49.6	.116
		633,796.375	-1.547	.0606			
639,796.375	152				.1484	148.8	.087
		645,796.375	-0.808	.2090			
651,796.375	270				.2631	263.1	.181
		657,796.375	-0.069	.4721			
663,796.375	268				.2765	276.5	.261
		669,796.375	0.669	.7486			
675,796.375	169				.1721	172.1	.056
		681,796.375	1.409	.9207			
687,796.375	69				.0635	63.5	.476
		693,796.375	2.148	.9842			
699,796.375	15				.0139	13.9	.087
		705,396.375	2.886	.9981			
711,796.375	1				.0018	1.9	.426
							6.39

$v = 10 - 1 - 2 = 7$. $\alpha = .01$. $x^2_{7,.99} = 18.475$. Fit is satisfactory.

APPENDIX D

COMPUTER OUTPUT: CHI-SQUARE TEST

FOR GENERAL EXPRESSION 4

```
**********************************
* PROPAGATION OF ERRORS ANALYSIS *
**********************************

FORMULA MEAN VALUE           1.0699

STANDARD DEVIATION =         0.0021

PERCENTAGE ERROR  =          0.0013

**********************************

     ************************
     *  PROBABILITY LIMITS  *
     ************************

1.  99.7%   (         1.0636,          1.0762)

2.  99.5%   (         1.0639,          1.0759)

3.  99.1%   (         1.0642,          1.0756)

     ***********************
     * SIMULATION ANALYSIS *
     ***********************
FORMULA MEAN VALUE             1.0699
STANDARD DEVIATION             0.0022


      *******************
      * FREQUENCY TABLE *
      *******************

   * INTERVAL * FREQUENCY *

      1.0614
      1.0626-          2.

      1.0626
      1.0638           1.

      1.0638
      1.0650-          8.

      1.0650
      1.0662-         26.

      1.0662
      1.0674-         75.

      1.0674
      1.0686         149.

      1.0686
      1.0698         226.

      1.0698
      1.0710-        196.

      1.0710
      1.0722         165.

      1.0722
      1.0734          97.

      1.0734
      1.0746          35.

      1.0746
      1.0758-         17.

      1.0758
      1.0770-          3.

      1.0770
      1.0782           0.

      1.0782
      1.0794           0.
```

```
                              HISTOGRAM OUTPUT OF SIMULATION
                              ******************************

                                         FREQUENCY
                                      ( 1 DOT =  4.6)

                     .................................................*
              1.0614 *
                     *
                     *
              1.0626 *
                     *
                     *
              1.0638 **
                     .*
                     .*
              1.0650 ******
                     .    *
                     .    *
              1.0662 *****************
                     .               *
                     .               *
              1.0674 *********************************
                     .                               *
                     .                               *
              1.0686 **************************************************
                     .                                                *
                     .                                                *
              1.0698 **************************************************
                     .                                         *
                     .                                         *
              1.0710 *******************************************
                     .                                  *
                     .                                  *
              1.0722 ************************************
                     .                    *
                     .                    *
              1.0734 **********************
                     .      *
                     .      *
              1.0746 ********
                     .  *
                     .  *
              1.0758 ****
                     *
                     *
              1.0770 *
                     *
                     *
              1.0782 *
READY
```

TABLE IV

OBSERVED DISTRIBUTION OF 1000 RANDOM VALUES FOR THE MASS FLOW RATE EQUATION
CLASS FREQUENCIES CALCULATED UNDER HYPOTHESIS OF NORMALITY,
AND GOODNESS OF FIT TESTED BY CHI-SQUARE

(1) Mid Value x	(2) Observed Freq., f_i	(3) Class Bounds, x	(4) Standard Var. Z	(5) Area Below Z in (4)	(6) Area Between	(7) Calcul. Freq., F_i	(8) Contr. to x^2
1.0620	2				.0005	.5	4.5
		1.0626	-3.32	.0005			
1.0632	1				.0023	2.3	.73
		1.0638	-2.77	.0028			
1.0644	8				.0101	10.1	.44
		1.0650	-2.23	.0129			
1.0656	26				.0336	33.6	1.72
		1.0662	-1.68	.0465			
1.0668	75				.0806	80.6	.39
		1.0674	-1.14	.1271			
1.0680	149				.1505	150.5	.01
		1.0686	-0.591	.2776			
1.0692	226				.2064	206.4	1.86
		1.0698	-0.045	.4840			
1.0704	196				.2075	207.5	.64
		1.0710	.50	.6915			
1.0716	165				.1593	159.3	.20
		1.0722	1.045	.8508			
1.0728	97				.0933	93.3	.15
		1.0734	1.591	.9441			
1.0740	35				.0397	39.7	.56
		1.0746	2.14	.9838			
1.0752	17				.0125	12.5	1.6
		1.0758	2.68	.9963			
1.0764	3				.0037	3.7	.13
							12.9

$v = 13 - 1 - 2 = 10$. $\alpha = .01$. $x^2_{10,.99} = 23.209$. Fit is satisfactory.

VITA

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